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"ANOMALOUS" NONLINEAR WAVE PHENOMENA IN A SUPERSONIC BOUNDARY LAYER

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Nonlinear evolution of high-amplitude periodic disturbances in a boundary layer on a flat plate for Mach number M = 2 *is studied. An anomalous downstream evolution of the disturbances is found, quasi-two-dimensional disturbances being most unstable. The obtained phase velocities of the waves are* 30–40% *greater than the phase velocities of the Tollmien–Schlichting waves. The nonlinear evolution of vortex waves is accompanied by an increase in steady disturbances from the source of controlled vibrations. High-frequency disturbances decay, and a periodic wave train degenerates downstream into a quasiharmonic wave train.*

Introduction. Experiments [1] conducted for a comparatively low amplitude of initial disturbances showed that the mechanism of nonlinear interaction of unsteady waves in a supersonic boundary layer is the parametric resonance. The major portion of energy of subharmonic disturbances belongs to perturbations with wave-inclination angles of about 80°. In experiments [2, 3], however, it was found that an increase in the amplitude of initial disturbances leads to nonlinear amplification of quasi-two-dimensional subharmonic oscillations. Commonly accepted concepts of the laminar–turbulent transition in the boundary layer at supersonic velocities associate it with amplification of three-dimensional disturbances [1, 4–7]. Therefore, the excitation of quasi-two-dimensional waves in a supersonic boundary layer is an exception from the general case, and it can be called anomalous. The essence of the approach used in [2, 3] to study nonlinear stability of a supersonic boundary layer is the use of the amplitude of initial disturbances as a parameter in solving the problem. The use of this approach is justified by experience of studying nonlinear stability of an incompressible boundary layer. It is known that there are two types of the laminar–turbulent transition in an incompressible boundary layer in controlled experiments: N- and K-type. The N-type transition occurs when the amplitudes of initial disturbances are small, and the K-type transition is observed with increasing amplitude [4, 8–10]. Results of experiments conducted for the maximum allowable amplitude of initial disturbances for a source of controlled oscillations are presented below. These experiments continue the tests started in [3].

Test Conditions. The experiments were conducted in the T-325 supersonic wind tunnel of the Institute of Theoretical and Applied Mechanics, Siberian Division of the Russian Academy of Sciences, for Mach number M = 2 and unit Reynolds number $\mathrm{Re}_1 = 6.5 \cdot 10^6$ m^{-1}. The flow velocity in the test section $U = 504$ m/sec. The model was a flat steel plate of length 450 mm, width 200 mm, and thickness 10 mm. The angle of the leading edge was 14°30′, and its thickness was 0.02 mm. The plate was mounted in the central plane of the test section of the wind tunnel at zero incidence. Controlled perturbations were introduced into the boundary layer by a generator of localized artificial disturbances whose construction is based on a spark discharge in the chamber [11]. Artificial disturbances were introduced into the boundary layer through an aperture 0.42 mm in diameter in the working surface of the model; the ignition frequency of the discharge was 20 kHz. The source coordinates were $x = (38 \pm 0.25)$ mm and $z = 0$ (x is the distances from the leading edge of the model and $z = 0$ corresponds to the model centerline).

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The fluctuations were measured by a constant-temperature hot-wire anemometer with a 1 : 10 ratio of the bridge arms and frequency range to 500 kHz and tungsten-wire probes of diameter 5 μm and length 0.95 mm. The measurements were conducted in the layer of maximum natural fluctuations across the boundary layer, i.e., relative to y/δ (δ is the boundary-layer thickness). The probe on a traversing gear moved along the coordinates x, y, and z. The probe position was determined within 0.1 mm along the x and z coordinates and 0.01 mm along the y axis. When the probe was moved along the x axis, the voltage in the diagonal of the hot-wire anemometer bridge was kept constant owing to the displacement of the probe in the y direction, which is equivalent to measurements with $\rho u =$ const (ρu is the mass flow) and $y/\delta =$ const. When the probe was moved in the z direction, the measurements were performed for $x =$ const and $y =$ const. The overheat ratio of the wire was 0.8, and the measured disturbances corresponded to mass-flow fluctuations.

The mean and fluctuating characteristics of the flow were measured by an automated data acquisition system similar to that described in [11]. The fluctuating voltage (within the frequency range to 350 kHz) from the hot-wire anemometer was recorded on a computer through a 12-digit analog-to-digital converter (ADC) with a frequency 750 kHz. The ADC was triggered simultaneously with the generator, which set the frequency of disturbances introduced. The accuracy of ADC triggering was better than 0.2%. To improve the signal-to-noise ratio, the signal was simultaneously summed over 128 realizations, the realization length being 1024 points. The mean voltage from the hot-wire anemometer was recorded on a computer through input registers connected to a Shch1516 voltmeter. The frequency harmonics were determined using a discrete Fourier transform of the averaged oscillograms. The length of each realization was 900 points of the initial signal.

The spectral treatment of experimental data was performed by a discrete Fourier transform

$$e_{f\beta}(x,y) = \frac{1}{T}\sum_{j,k} e(x,z_j,y,t_k)\exp\left(-i[\beta z_j - f t_k]\right),$$

where $e(x,z_j,y,t_k)$ is the fluctuating signal from the hot-wire anemometer averaged over realizations, T is the time of one realization, β is the dimensional spanwise wavenumber, and f is the frequency. The amplitude and phase of the disturbances were found from the discrete Fourier transform from the formulas

$$A_{f\beta}(x,y) = \{\mathrm{Real}^2[e_{f\beta}(x,y)] + \mathrm{Im}^2[e_{f\beta}(x,y)]\}^{0.5},$$

$$\Phi_{f\beta}(x,y) = \arctan\{\mathrm{Im}[e_{f\beta}(x,y)]/\mathrm{Real}[e_{f\beta}(x,y)]\}.$$

Analysis of the Results Obtained. Under our experimental conditions with excitation of controlled disturbances, it was found that the laminar–turbulent transition on the model occurred 20% closer to the leading edge along the streamwise coordinate ($x_{\rm tr} = 205$ mm, Reynolds number $\mathrm{Re}_{\rm tr} = \sqrt{\mathrm{Re}_1 x_{\rm tr}} = 1150$) than for natural disturbances ($x_{\rm tr} = 250$ mm and $\mathrm{Re}_{\rm tr} = 1280$).

The evolution of modulated wave packets was measured in cross sections along the transverse coordinate z for $x = 60$, 70, 90, and 110 mm, $\mathrm{Re} = \sqrt{\mathrm{Re}_1 x} = 624$, 674, 765, and 846. Figure 1a and b shows z-oscillograms of controlled disturbances for $x = 60$ mm and $x = 110$ mm, respectively. The amplitude scale of the hot-wire signal here corresponds to a 12-digit representation of the ADC. "Thorn"-type oscillograms are observed in the center of the wave packet for $x = 60$ mm, whereas the oscillograms of controlled disturbances measured for $x = 110$ mm are almost sinusoidal with a period of 100 μsec. Thus, it is experimentally demonstrated that the downstream evolution of a periodic wave train leads to quasiharmonic disturbances.

It follows from the two-dimensional character of the boundary-layer flow on a flat plate that this flow should be uniform along z. This fact was experimentally verified many times (see, for example, [6, 11]). Since the amplitude of initial disturbances was large, local changes (deceleration and displacement) of the mean flow in the center of the wave packet occurred. The mean-voltage defect in the center of the wave packet was about 0.05 V for $x = 60$ mm and about 0.15 V for $x = 110$ mm. Considering the magnitude of the mean-voltage defect as a dimensional amplitude of steady disturbances introduced by a point source, we can state that the amplitude of steady disturbances increases downstream. In the measured distributions, the z-width of an unsteady wave packet coincided with the z-width of the mean-voltage defect (of a steady disturbance). Figure 2 shows distributions of the mean voltage E and the amplitude of fluctuations A_f of frequency $f = 10$ kHz and

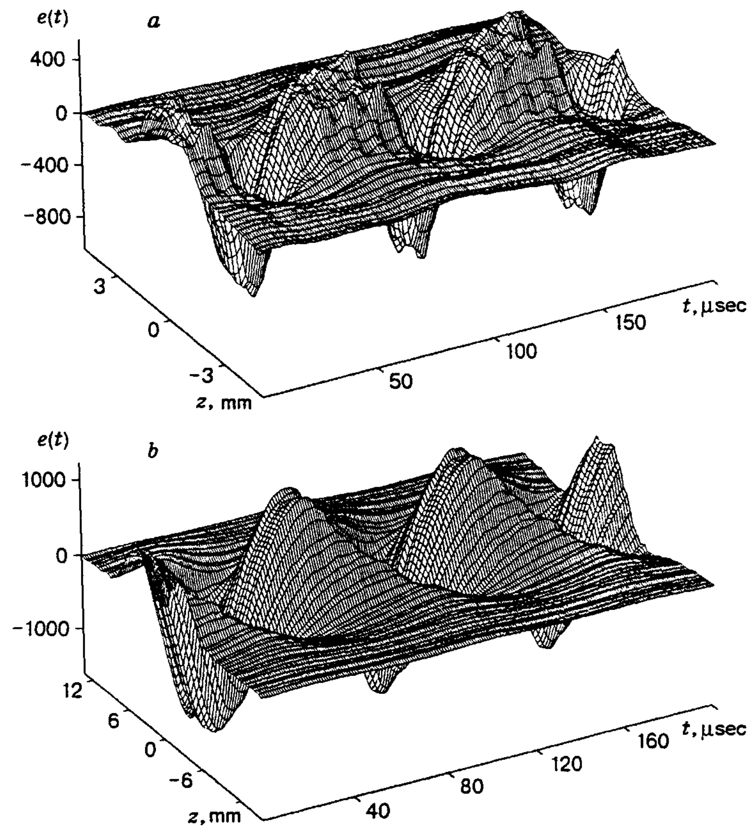

Fig. 1

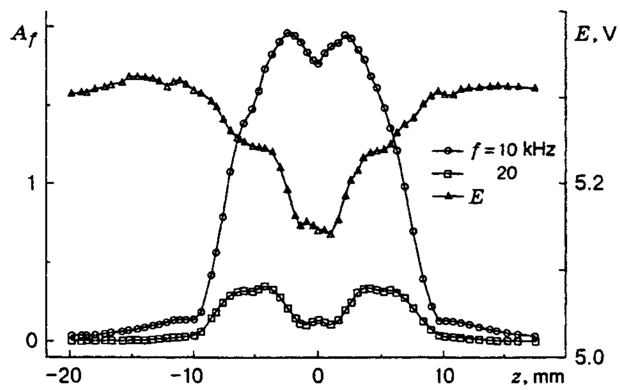

Fig. 2

$f = 20$ kHz (frequency parameter $F = 2\pi f/(\mathrm{Re}_1 U_\infty) = 0.192 \cdot 10^{-4}$ and $0.384 \cdot 10^{-4}$) along z for $x = 110$ mm. The amplitude is presented here in percentage from the dimensionless magnitude of fluctuations $A_f = e_f/E$.

The excited disturbances distorted the mean-velocity profile across the boundary layer so that the boundary-layer thickness increased. For example, for $x = 110$ mm, the boundary-layer thickness in the center of the wave packet is almost twice the boundary-layer thickness δ observed under natural conditions without controlled disturbances.

The profiles of controlled disturbances had maxima along y. For disturbances of frequency 10 kHz for $x = 110$ mm and $z = 0$, this maximum was located at a distance of 0.6 mm from the model surface, and the maximum of disturbances of frequency 20 kHz was located twice as far from the wall. In our experiments, the 10-kHz disturbances were measured near the maximum in the y distribution, and the 20-kHz disturbances were measured below the maximum. This is caused by the fact that the coordinate y = const, which was

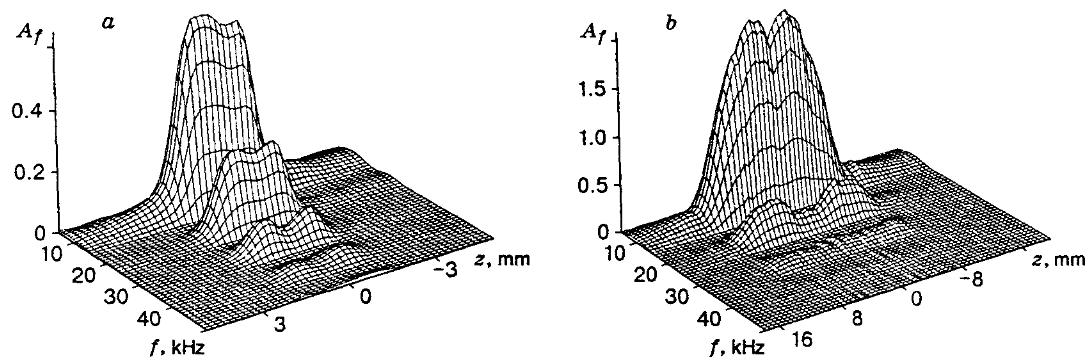

Fig. 3

used for transverse measurements, was chosen from the maximum of natural fluctuations across the boundary layer.

We cannot state, however, that the maximum of controlled disturbances in the layer is located at an equal distance from the model surface as the transverse coordinate z is varied. To check this, multiple measurements of the fluctuating profiles at each point along z are needed. After obtaining detailed profiles of fluctuations, one has to perform a wave analysis of the z-distributions for different values $y = \text{const}$ and then construct profiles of fluctuations corresponding to planar waves. This is a very labor-consuming and time-consuming work, which has not been performed experimentally yet.

The experimental results on evolution of a nonlinear wave packet were analyzed similar to [3]. The distributions of the amplitudes A_f of controlled oscillations in the transverse direction for $x = 60$ mm and $x = 110$ mm are shown in Fig. 3a and b, respectively; the corresponding amplitude spectra of the disturbances $A_{f\beta}$ are plotted in Fig. 4a and b. The results are presented in the frequency range from 5 to 50 kHz ($F = 0.96 \cdot 10^{-5}$–$0.96 \cdot 10^{-4}$), since high-frequency oscillations were not observed. Note that the total width of the wave train Δz increased from 4 mm at $x = 60$ mm to 20 mm at $x = 110$ mm, and the corresponding angle of spanwise spreading of the wave train to z was $\pm 9°$, which is greater by a factor of 1.5 than in the case of linear evolution [6] and by a factor of 6 than in the case of nonlinear evolution of the wave packet at M = 3 [3].

As follows from Fig. 4a, the wave spectra at all frequencies have a maximum at $\beta = 0$, i.e., two-dimensional waves prevail in the initial spectra of the disturbances. This result was previously obtained by Kosinov et al. [5] at M = 4, where the initial wave spectra had a maximum at $\beta = 0$ and frequency of 20 kHz. Nevertheless, subsequent linear evolution of disturbances in the boundary layer at M = 4 led to transformation of the initial spectra with a maximum at $\beta = 0$ to disturbances with a maximum in the spectra at $\beta = 0.4$–0.7 rad/mm, i.e., to a strong increase in three-dimensional disturbances, which was predicted by linear stability theory. This does not occur in the present experiments, however; here, a strong growth of disturbances at the frequency of 10 kHz (almost a tenfold increase from $x = 60$ mm to $x = 110$ mm) is observed, and the two-dimensional character of the wave spectra in this range is retained (see Fig. 4). An increase in disturbances of frequency of 20 kHz (by a factor of four on the average) is observed as x changes from 60 to 90 mm. Only at $x = 110$ mm do three-dimensional waves ($\beta > 0$) begin to prevail in the wave spectra of disturbances of frequencies of 20 and 30 kHz; as a whole, the increase in three-dimensional disturbances is more significant than in two-dimensional ones.

From linear stability theory and experiments [6] conducted for small perturbations, it follows that the amplification of disturbances of frequency of $f = 20$ kHz in this region of the boundary-layer flow should be more intense than the amplification of disturbances with $f = 10$ kHz, and three-dimensional rather than two-dimensional waves are most unstable. However, the opposite situation is observed in the experiments described. The results presented have no analogs with the earlier data or with the results of hydrodynamic stability theory. Thus, the observed degeneration of the modulated wave packet into a harmonic packet with prevailing growth of two-dimensional disturbances can be considered as a phenomenon caused by nonlinearity of the process.

On the basis of the phase spectra obtained, we evaluated the phase velocities of the disturbances:

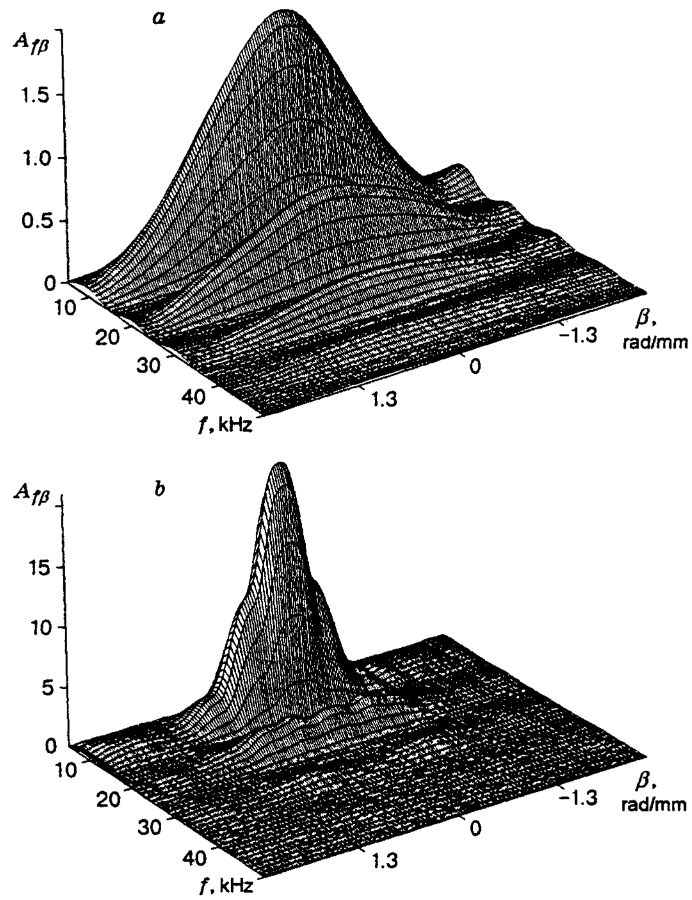

Fig. 4

$C(\chi) = (\Delta x/\Delta\Phi_{f\beta})F\mathrm{Re}_1$, where χ is the angle of inclination of the wave vector to the flow direction. For disturbances of frequency of 10 kHz, these results are plotted in Fig. 5 for x = 60–70 mm (curve 1) and x = 90–110 mm (curve 2). The data obtained as $C(\chi)$ have a unique character and have not been observed previously in experiments on stability in a supersonic boundary layer. For example, the theoretical dependence $C(\chi)$ for $f = 20$ kHz obtained in calculations based on linear stability theory (cf. experimental and numerical results in [6]) has a typical maximum at $\chi = 0$ and a minimum at $\chi \approx \pm 55°$. In our experiments with $f = 10$ and 20 kHz, the phase velocities increase with increasing angle of wave inclination; the numerical values of phase velocities are greater than those of the intrinsic waves of a supersonic boundary layer by 30–40% on the average.

A harmonic character of evolution of the wave train was observed in the present experiments until $x = 150$ mm (Re = 987); this was the greatest value of the x coordinate for which the measurements were taken. The distribution of fluctuations across the boundary layer obtained for $x = 140$ mm (Re = 954) shows that the amplitude of the wave train rapidly decreases with distance from the model, which is typical of vortex disturbances. Near the outer edge of the shear layer, a distortion of the sinusoid character of the disturbances to a smoothed saw-tooth profile and then back to a sinusoid shape was observed. This change in disturbance oscillograms corresponds to the appearance of high-frequency oscillations at the outer edge of the boundary layer.

Owing to the high resolution of the ADC (12 bit), we measured the evolution of the disturbances outside the boundary layer at $z = 0$. As follows from the data obtained, the amplitude of 20-kHz disturbances decreases downstream from $x = 100$ mm to $x = 150$ mm in the external part of the flow, and 10-kHz disturbances increase. The phase velocities estimated from the dependences $\Phi_{f\beta}(x)$ yield values close to those

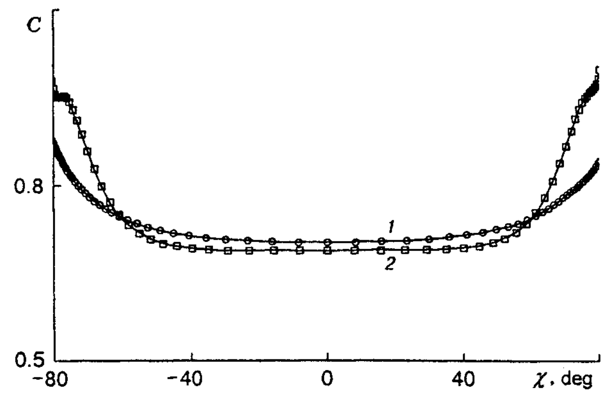

Fig. 5

presented in Fig. 5. The wave spectra as functions of β of oscillations of frequency of 10 and 20 kHz had a maximum at $\beta = 0$ in the external part of the flow, and phase synchronism of oscillations was observed at $|\beta| < 0.25$ rad/mm.

Conclusions. In contrast to the commonly accepted notion about the three-dimensional nature of the laminar–turbulent transition, amplification of two-dimensional disturbances in the boundary layer was observed in studying the evolution of a nonlinear wave packet at M = 2. The amplification of two-dimensional disturbances was called anomalous in the sense of the exclusive character of this process at moderate supersonic Mach numbers.

The strong growth of quasi-two-dimensional disturbances is, apparently, related to the growth of a steady vortex, which was generated in our experiments by a source of controlled disturbances. Since the instability observed is two-dimensional, there are reasons to believe that the phenomenon found in these experiments can be used to stabilize the flow in a supersonic boundary layer. However, a decrease in the transition Reynolds number Re_{tr} by 20% as compared with Re_{tr} for natural disturbances was observed in our experiments, i.e., destabilization of the flow. We suppose that flow destabilization could occur due to the adverse effect of interaction between the Mach waves and the boundary layer on evolution of the disturbances (see, for example, [12]). Another fact in favor of the stabilizing effect is the degeneration of high-frequency harmonics resulting from downstream evolution of the disturbances. The disturbances under study are vortex disturbances whose phase velocities are greater by 30–40% than those of the intrinsic waves of a supersonic boundary layer.

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